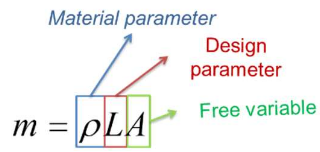
# 1. Using Ashby Plots *[full solutions]*

Let us consider a beam with square cross-section of area $A$.

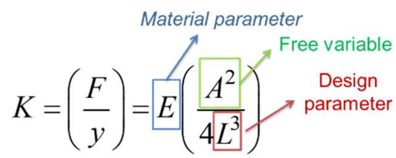
1. What *material properties* should we optimize to minimize the mass of a beam of length L and achieve the maximum stiffness in bending? Follow the method provided below (Steps 1-3).

**Step 1.**

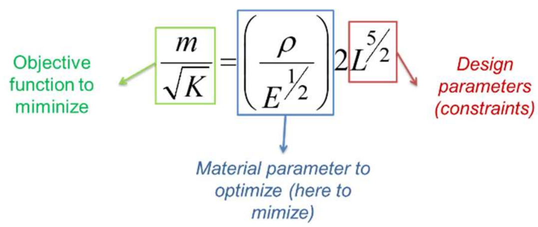
- Minimizing mass




- Bending stiffness of a square cantilever beam




**Step 2.**




The problem is equivalent to maximizing

$$C = \frac{\sqrt{E}}{\rho}$$

where C is arbitrarily chosen as a function to the final design requirement.

**Step 3.** To visualize it as a line on an Ashby log-log plot, we express it as follows:

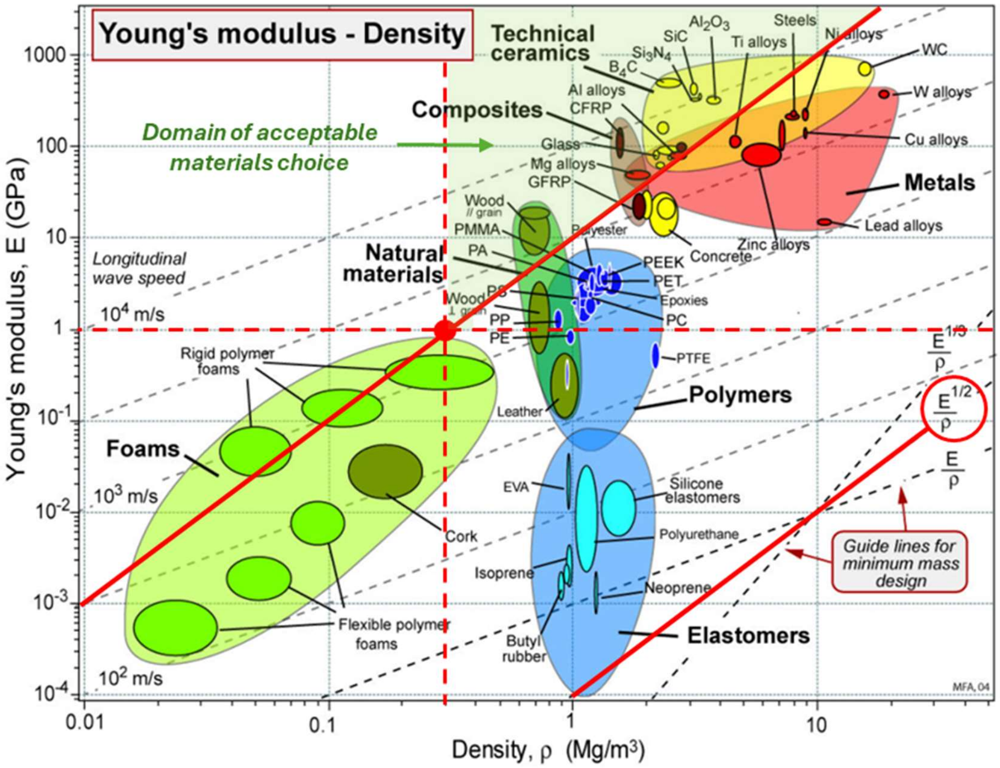
$$log(E) = 2[log(\rho) + log(C)] \Leftrightarrow y = ax + b$$

This is equivalent to a line on the log-log plot that can be used to define a frontier between materials that are suitable for this particular problem or not.

2. Which material would you select considering this problem? We will assume as requirements that the minimum value for the modulus should be $1\ \mathrm{GPa}$ and that the material should have a density higher than $0.3\ \mathrm{Mg/m^3}$ (or $300\ \mathrm{kg/m^3}$). Follow Step 4.

**Step 4.** We consider the Young's modulus versus density plot (**Figure 3**). To fix the frontier, we use the two additional numerical requirements provided in the exercise.

$$y = ax + b \Leftrightarrow log(E) = 2 \cdot log(\rho) + 2 \cdot log(C) = 2 \cdot log(\rho) + 10$$




We conclude that a wide range of materials may satisfy the problem, ranging from natural materials such as wood to composites or technical ceramics like SiC. As the ratio should be maximized, we need to select a material above the red line, in the domain of acceptable materials. Note that the final choice of material will depend on additional information such as cost, that defines the optimization problem. We also may think about the application operating temperature, resistance to fatigue, recyclability, etc.